

## Scalar Mesons and Chiral Symmetry

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It is the purpose of the present manuscript to emphasize those aspects that make the scalar sector with vacuum quantum numbers rather unique. Chiral symmetry is the basic tool for our study together with a resummation of Chiral Perturbation Theory (CHPT) that stresses the role of unitarity but also allows one to include explicit resonance fields and to match with the CHPT expansion at low energies.

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### §1. Introduction

In the  $J^{PC} = 0^{++}$  sector hadrons really interact strongly. In fact, one can find in the literature a consensus with respect to the strong unitarity corrections (rescattering effects) that take place in this sector<sup>1)-7)</sup>. As a matter of fact, this strong rescattering usually involves different channels with very pronounced cusp structures at the opening of the heavier thresholds coinciding with the presence of resonance states: For the  $I = 0$  case one has the  $f_0(980)$  and the  $K\bar{K}$  channel, for the  $I = 1$  the  $a_0(980)$  and the  $K\bar{K}$  threshold and finally in the  $I = 1/2$  one has the  $K\eta'$  channel and the  $K_0^*(1430)$  resonance. Of course, in these cases simple Breit-Wigner forms for resonances are not justified and even the appearance of dynamical resonances originated by the residual but, in this case, strong meson-meson interactions can take place<sup>1), 6)-8)</sup>. In any case, these strong unitarity corrections tend to mask the resonance properties by making some of them very wide and also by providing large backgrounds that have to be pinned down rather precisely.

In connection with the previous discussions, the appearance of large violations of the Okubo-Zweig-Izuka (OZI) rule in the  $0^{++}$  sector can be expected. The only general and well founded explanation of the OZI rule is large  $N_c$  QCD<sup>9)</sup> since OZI violating graphs imply the presence of an extra quark loop diagram and these contributions are suppressed by a factor  $1/N_c$ . In the large  $N_c$  limit mesons do not interact with each other<sup>9)</sup>, establishing in this way the basis for a weakly coupling theory of the strong interactions in terms of hadronic degrees of freedom in which the leading contributions come from tree-level diagrams (local and pole terms). This picture is, to a large extend, highly successful in the vector and tensor channels. However, in the  $0^{++}$  sector loops, which are responsible for the unitarity of the  $S$ -matrix, are strongly enhanced and then it is natural to expect a departure from the

large  $N_c$  scenario. For instance, it is clear that the  $0^{++}$  spectrum is not dominated by ideally mixed  $\bar{q}q$  nonets, being the latter a consequence both of large  $N_c$  and the OZI rule. Notice that the OZI rule is one of the key ingredients in simple quark models and has been usually advocated as a way to pin down values for low energy constants in Chiral Perturbation Theory<sup>10)</sup>.

## §2. Crossed channel dynamics and lowest order CHPT

In this section we want to point out the facts which drive the phenomenological behavior of the  $I = 0, 1$  and  $1/2$   $0^{++}$  channels. The first one has its roots in a violation of the expected large  $N_c$  results and the second is just a requirement of chiral symmetry.

### 2.1. Crossed channel dynamics

In ref.<sup>7)</sup> the unphysical cut contributions, due to crossed channel dynamics, were estimated making use of the results of ref.<sup>11)</sup> in which the next-to-leading order CHPT amplitudes were supplied with the exchange of explicit resonance fields from the chiral symmetric Lagrangians of ref.<sup>12)</sup>. In this way, the range of applicability of chiral constraints was enlarged up to around 0.8 GeV. From the diagrams considered in ref.<sup>11)</sup>, one isolates those corresponding to loops and to the exchange of resonances both in the crossed  $t$ - and  $u$ - channels, figs.1a,b respectively. For further details we refer to section V of ref.<sup>7)</sup>. The important result was that up to 0.8 GeV this set of diagrams amounts to just of a few percent of the sum of the  $s$ -channel exchange of resonances plus the lowest order CHPT amplitude, figs.1c,d respectively. This result has been recently corroborated in ref.<sup>13)</sup>.

As discussed in ref.<sup>7)</sup> the smallness of the unphysical cut contributions in the physical region is a consequence of a large cancelation between the crossed loops and the crossed exchange of resonances, otherwise these contributions would be important. That this cancelation takes place is a clear signal of large  $N_c$  violation in these channels because loop physics is suppressed by an extra  $1/N_c$  factor with respect to the tree-level exchange of vector plus scalar resonances.

### 2.2. Lowest order CHPT

In ref.<sup>7)</sup> the general structure of a partial wave amplitude when the unphysical cuts are discarded was shown to be:

$$T = \left[ \sum_i \frac{\gamma_i}{s - s_i} + a^L + g(s) \right]^{-1} \quad (1)$$

where the sum extends over the CDD poles<sup>14)</sup>,  $a^L$  is a leading subtraction constant in the large  $N_c$  counting as  $\mathcal{O}(N_c)$ , and  $g(s)$  is the two meson loop function depicted in fig.2 and is  $\mathcal{O}(N_c^0)$ . For the case of equal meson masses  $m$ ,  $g(s)$  is given by:

$$g(s) = a^{SL}(\mu) + \frac{1}{(4\pi)^2} \left[ \log \frac{m^2}{\mu^2} + \sigma(s) \log \frac{\sigma(s) + 1}{\sigma(s) - 1} \right] \quad (2)$$

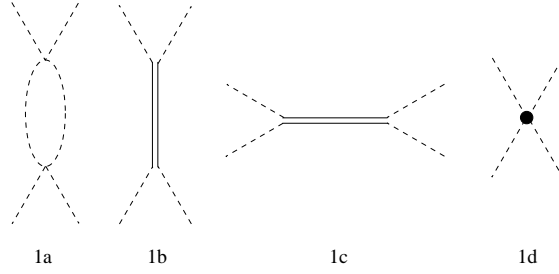


Fig. 1. Fig.1a:  $t$ - and  $u$ -channel crossed loops at  $\mathcal{O}(p^4)$  in CHPT. Fig.1b:  $t$ - and  $u$ -channel exchange of scalar and vector resonances. Fig.1c:  $s$ -channel exchange of scalar and vector resonances. Fig.1d: Lowest order CHPT.

In the following we will take  $\mu = M_\rho$  with  $M_\rho$  the mass of the  $\rho$  meson. However, it should be clear that our results are scale independent since any change in the scale  $\mu$  of  $g(s)$  is reabsorbed by a change in  $a^{SL}(\mu)$ .

Notice that a CDD pole corresponds to a zero of the amplitude  $T$  since it is a pole in the *denominator*. Interestingly, due to the Goldstone boson nature of the pions, the presence of the Adler zeroes follows and hence, at least, one needs a CDD pole per partial wave (notice that  $a^L$  can be considered as a CDD pole at infinity). The position of this pole ( $s_0$ ) and its residue ( $\gamma_0$ ) is just the information contained in the lowest order CHPT amplitudes. Of course, both the position and the residue are given to the corresponding lowest order.

Let us go now to consider the consequences of the presence of the Adler zeroes by restricting the sum over the CDD poles in eq. (1) to only the one that corresponds to the Adler zero for each particular partial wave. We will consider below the stability of the results under the inclusion of higher CDD poles.

We will also discuss simultaneously the S- and P-wave  $\pi\pi$  partial amplitudes in order to make manifest the striking differences between both cases.

To reproduce the  $\pi\pi$  P-wave scattering data, which is clearly dominated by the presence of the  $\rho$  resonance, one needs<sup>7)</sup>:

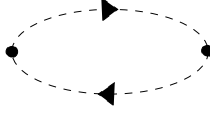
$$\begin{aligned} a^L &\simeq -\frac{6f_\pi^2}{M_\rho^2} \simeq -9 \times 10^{-2} \\ a^{SL} &= 0 \end{aligned} \tag{3}$$

where the analytic expression for  $a^L$  follows from Vector Meson Dominance<sup>7)</sup>.

On the other hand, in order to describe the data for the  $\pi\pi$   $I = 0$  S-wave below 0.8 GeV, where two pions are still the relevant intermediate states, one has<sup>7)</sup>:

$$\begin{aligned} a^L &= 0 \\ a^{SL} &\simeq -5 \times 10^{-3} \end{aligned} \tag{4}$$

From these values it follows as well the presence of the  $\sigma$  pole by applying eq. (1)<sup>7), 8)</sup>.

Fig. 2. Two meson loop function  $g(s)$ .

For the  $\pi\pi$  P-wave the Adler zero is just located at threshold while in the second case, for the S-wave, is close to  $m_\pi^2/2$ . The differences between eqs. (3) and (4) are really astonishing. To further sharpen these statements, notice that while one can reabsorb  $a^{SL}$  in eq. (4) just by a change of order one in the scale  $\mu$ , this is not possible for  $a^L$  in eq. (3). In fact, the necessary change in the scale  $\mu$  in this case is:

$$\mu \rightarrow \exp(-8\pi^2 a^L) \mu \simeq 1 \text{ TeV.} \quad (5)$$

while any ‘natural’ scale  $\mu$  should be around 1 GeV.

This result tells us that the  $\rho$  is a preexisting state that cannot be generated by loop physics, that is, its mass is  $\mathcal{O}(N_c^0)$ . In contrast, the  $\sigma$  meson generated by applying the results of eq. (4) to eq. (1) can be considered as a dynamical resonance generated through the iteration of the lowest order CHPT amplitude. In this case, its mass counts as  $f_\pi^2 = \mathcal{O}(N_c)$  and hence, in the large  $N_c$  limit, the pole disappears by moving to infinity.

Alternatively, we can also present the previous discussion by saying that if we wanted to generate a pole with a mass around 0.8 GeV in the  $\pi\pi$  P-wave amplitude just by the iteration of the lowest order CHPT amplitude ( $a^L = 0$ ), we would need a scale of around 1 TeV whereas in the S-wave the resulting scale is of the order of 1 GeV. This difference arises because of the residues of the CDD poles associated to the Adler zeroes. While in the S-wave one has  $f_\pi^2$  in the P-wave the residue is  $6f_\pi^2$ , that is, there is an enhancement of a factor 6 in the later case with respect to the former. Because the scale  $\mu$  only appears logarithmically in  $g(s)$  all these factors of difference imply an exponential scaling in  $\mu$ . Of course, these residues are just consequence of chiral symmetry.

This interpretation about the dynamical origin of the  $\sigma$  meson was tested in ref. <sup>7)</sup> by allowing the presence of explicit resonances, or equivalently, incorporating more CDD poles. The result was that after fitting the present experimental data, no preexisting resonance appeared in the  $I = 0$  S-wave  $\pi\pi$  that could be related to the  $\sigma$  pole.

### §3. SU(3) and Final State Interactions

#### 3.1. SU(3) related channels

The results discussed in the previous section were generalized to the SU(3) case in ref. <sup>7)</sup> and an improved and very detailed study of the S-wave  $K\pi$  scattering has been recently given in ref. <sup>13)</sup>.

In ref. <sup>7)</sup> the coupled channel version of eq. (1) was also derived and used to further investigate the whole set of SU(3) connected  $I = 0, 1, 1/2$  S-wave meson–meson

partial amplitudes from threshold up to about 1.3–1.4 GeV. For higher energies multiparticle states cannot be further neglected.

In that paper together with the Adler zeroes previously discussed, the inclusion of two nonets of preexisting scalar resonances with masses below 1.5 GeV was allowed. However, after fitting phase shifts and inelasticities, the couplings of one of these scalar nonets were compatible with zero and the fit only required the presence of one scalar octet with a mass around 1.4 GeV and of a singlet around 1 GeV. For further details see ref. <sup>7)</sup>.

The resonance content of the solutions was also studied <sup>7)</sup> and two sets of scalar resonances was observed. The octet of preexisting resonances around 1.4 GeV gives rise to eight resonance poles with masses very close to the physical resonances  $f_0(1500)$ ,  $a_0(1450)$  and  $K_0^*(1430)$  <sup>16)</sup>. The singlet resonance around 1 GeV evolves continuously from its bare pole position to the final one giving rise to a contribution to the  $f_0(980)$  resonance (this can be seen by inserting gradually  $g(s)$  in eq. (1), multiplying it by a factor  $\lambda$  which takes values between 0, tree-level, and 1, final result).

However, the presence of extra resonance poles that do not originate from the set of the preexisting ones was also observed. These comprise the  $a_0(980)$  pole with  $I = 1$ , the  $\kappa$  with  $I = 1/2$  and two poles in the  $I = 0$  case: the  $\sigma$  and a very important contribution to the  $f_0(980)$  due to the  $K\bar{K}$  threshold. All these poles originate just by iterating the lowest order CHPT amplitudes, as discussed in the previous section with respect to the  $\sigma$  meson. In fact, it was also observed in ref. <sup>7)</sup> that when moving continuously to the SU(3) limit these poles bunch together in a degenerate octet plus a singlet.

Ref. <sup>13)</sup> was devoted to a thorough study of the S-wave  $K\pi$  scattering amplitudes. For the  $I = 1/2$  a description of the data was accomplished up to about 2 GeV. The input considered in this reference is an improved one with respect to that used in ref. <sup>7)</sup>. The differences are: 1) Unphysical cut contributions were included by considering crossed exchange of vector and scalar resonances and crossed loops calculated at  $\mathcal{O}(p^4)$  in CHPT<sup>\*</sup>), 2) the  $K\eta'$  channel was included making use of a combined chiral and  $1/N_c$  expansion <sup>15)</sup>:  $1/N_c \sim \mathcal{O}(p^2)$ . Notice that this state is a fundamental one in order to study properly the  $K_0^*(1430)$  resonance. Despite these improvements the pole of the  $\kappa$  resonance remains basically in the same position as the one found in ref. <sup>7)</sup>. Furthermore, in this latter reference a perturbative expansion in terms of the unphysical cuts contributions for the  $I=0, 1$  and  $1/2$  S-wave meson–meson amplitudes was argued and the overall agreement between this reference and ref. <sup>13)</sup> gives further support to this point of view. We remind the reader what was already said in section 2.1.

In ref. <sup>13)</sup> it was also established that for the  $I = 3/2$   $K\pi$  S-wave scattering the unphysical cut contributions are not so small and one has to take care of them from the beginning. This is one of the reason why for this channel a description of the data ‘only’ up to around 1.3 GeV could be given. This situation can also be

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<sup>\*</sup>) In fact, a matching to the next-to-leading order  $K\pi$  amplitudes is given in ref. <sup>13)</sup> and at the tree level the approach is crossed symmetric.

applied to the  $I = 2$  S-wave  $\pi\pi$  scattering. Detailed discussions about the quality of the experimental data are also given in ref. <sup>13)</sup> where some of the experimental ambiguities are discarded.

### 3.2. Final State Interactions

As pointed out in ref. <sup>2)</sup> it is very interesting to complement the information coming directly from the strong interacting scattering data with that obtained by the study of the Final State Interactions (FSI) due to the strong interactions between the produced mesons.

It is then an important consistency check of the proposed solution to the S-wave puzzle to be able to reproduce in a systematic and unified way as many reactions as possible by taking care of the FSI. Along the years this program has been accomplished to a large extend and many reactions have been already reproduced:

- i)  $\gamma\gamma \rightarrow \pi^0\pi^0, \pi^+\pi^-, K^0\bar{K}^0, K^+K^-, \pi^0\eta$  ref. <sup>17)</sup>.
- ii)  $\phi \rightarrow \gamma K^0\bar{K}^0$  ref. <sup>18)</sup> and  $\phi \rightarrow \gamma\pi^0\pi^0, \gamma\pi^0\eta$  ref. <sup>19)</sup>.
- iii)  $J/\Psi \rightarrow \phi(\omega)\pi\pi, K\bar{K}$  ref. <sup>20)</sup>.

The  $T$ -matrix used to determine the FSI in the previous works is the one derived in ref. <sup>8)</sup> which is the limit case of ref. <sup>7)</sup> by keeping only the Adler zeroes. A detailed comparison between the  $T$ -matrices coming from refs. <sup>8), 21), 7)</sup> is given in section III of the review <sup>22)</sup>. It is interesting to note that all these processes are related by unitarity and chiral symmetry and this is the reason why such a unified description of all of them has emerged (it is not just a matter of adding more and more uncorrelated free parameters as done in ref. <sup>2)</sup>). In fact, in many of the papers collected in i), ii) and iii), one uses the well known result that if a  $T$ -matrix is written as  $N/D$  with  $D$  having only the unitarity or right hand cut, which is absent in  $N$ , then a production mechanism without unphysical cuts can be written\*) as  $R/D$  with  $R$  a function free of any cut, see also ref. <sup>20)</sup>. The  $R$  function is then fixed by requiring the matching with CHPT and/or making use of gauge invariance. Note that FSI are tremendously important in all the above collected processes and can modify by orders of magnitude the Born term contributions.

## §4. Chiral limit and chiral partners

It is an interesting exercise to consider formally the limit  $f_\pi \rightarrow 0$  in eq. (1). As it is well known  $f_\pi \neq 0$  is a necessary and sufficient condition to have spontaneous chiral symmetry breaking. Hence, by studying the case  $f_\pi \rightarrow 0$  we should obtain those features associated with the restoration of the chiral symmetry as for instance the appearance of particles degenerate in mass but with opposite parity. We will consider the SU(2) chiral limit,  $m_u = m_d = 0$  and  $m_s$  fixed.

It is clear that as  $f_\pi \rightarrow 0$  the chiral expansion no longer converges since the pion

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\*) Early considerations in these lines can be found in ref. <sup>23)</sup>.

interactions become non-perturbative. However, to a large extent, this is precisely the situation actually observed in the  $0^{++}$   $I = 0$ , 1 and 1/2 channels for the physical value of  $f_\pi$ . We have handled this problem by introducing the non-perturbative scheme discussed along the manuscript. In the remaining of the section we will assume that this scheme works as well in the limit  $f_\pi \rightarrow 0$ .

We have argued so far that a resummation of the CHPT series is very likely at the heart of the dynamical generation of the  $\sigma$  meson, eq. (1). Specializing this equation to the case of only one CDD pole corresponding to the required Adler zero, see section 2.2, we have the following expression for the  $I = 0$  S-wave  $\pi\pi$  amplitude:

$$T = \left[ \frac{f_\pi^2}{s} + g(s) \right]^{-1} \quad (6)$$

If we are interested in looking for a pole in  $T$  we have to consider  $g(s)$  in the second Riemann sheet, where it is given by:

$$g(s) = a^{SL} + \frac{1}{(4\pi)^2} \left( \log \frac{s}{\mu^2} + i\pi \right) \quad (7)$$

A pole of  $T$  is a zero of the denominator and hence one has the equation:

$$\frac{f_\pi^2}{s_0} = -a^{SL} + \frac{1}{(4\pi)^2} \left( \log \frac{\mu^2}{s_0} - i\pi \right) \quad (8)$$

with  $s_0$  the corresponding solution.

If  $f_\pi \rightarrow 0$  it is then convenient to write  $s_0 = \alpha f_\pi^2$  and therefore  $\alpha$  must fulfill:

$$\frac{1}{\alpha} + \frac{\log \alpha}{(4\pi)^2} = -a^{SL} + \frac{1}{(4\pi)^2} \left( \log \frac{\mu^2}{f_\pi^2} - i\pi \right) \quad (9)$$

Thus  $\alpha = \alpha(\log \frac{\mu^2}{f_\pi^2})$  and the former equation admits the solution  $\alpha(\log \frac{\mu^2}{f_\pi^2}) \rightarrow 0$  when  $f_\pi/\mu \rightarrow 0$ . \*) Then  $s_0 = \alpha f_\pi^2$  vanishes in the limit  $f_\pi \rightarrow 0$ .

As a result, in the limit of chiral symmetry restoration one has:

State	$\pi$	$\sigma$
Mass	$M_\pi = 0$	$m_\sigma = 0$
Parity	-1	+1

and hence the dynamically generated  $\sigma$  meson is the natural chiral partner of the pion.

The generalization of these considerations to the  $SU(3)$  case are straightforward and for instance in fig.9 of ref.<sup>7)</sup> the  $SU(3)$  set of dynamically generated poles are shown in the  $SU(3)$  chiral limit.

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\*) The same equation (8) is also found in the case of the strongly interacting Higgs sector when the massive states, that is, all the states except the vector gauge bosons, have a mass much larger than  $4\pi f_\pi \approx 3$  TeV, here  $f_\pi \equiv v \simeq 0.25$  TeV. In this case  $\log \frac{\mu^2}{f_\pi^2}$  also goes to infinite because the scale  $\mu$ , which is related to the mass of the massive particles, is much larger than  $f_\pi$ . This scenario is discussed in ref.<sup>24)</sup>.

## §5. Conclusions

In this manuscript, we have demonstrated how the lowest lying scalar resonances appear as a consequence of chiral symmetry together with unitarity. It is also shown that crossed channel dynamics is suppressed in the SU(3) related S-waves with  $I = 0$ , 1 and  $1/2$  in the pertinent energy region.

This picture has been tested already by post-dictions and also predictions of many production meson processes where FSI play a very important role giving rise to corrections of several orders of magnitude.

Finally we have addressed the limit of chiral symmetry restoration (both spontaneous as well as explicit) and we have seen that the dynamically generated  $\sigma$  meson is the required scalar degenerate in mass with the pions when chiral symmetry is restored.

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